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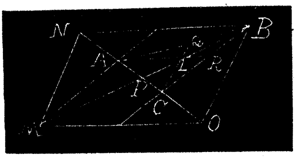
Draw the other diagonal NO. Trisect NO in A, C. The triangle ABC is the one required, since PB is evidently one of the medians given, and the other medians QC and AR are, respectively, equal to $\frac{1}{2}OB$ and $\frac{1}{2}NB$. This is clear, from the considerations of the similar triangles AOB and AQC ($AQ=\frac{1}{2}AB$, $AC=\frac{1}{2}AO$, $\therefore QC=\frac{1}{2}OB$), and NCB and ACR ($AC=\frac{1}{2}NC$, $RC=\frac{1}{2}BC$, $\therefore AR=\frac{1}{2}NB$).



CALCULUS.

90. Proposed by **ELMER SCHUYLER**, Reading, Pa.

Prove that the evolute of the logarithmic spiral is an equal logarithmic spiral. [From Byerly's *Integral Calculus*.]

I. Solution by **G. B. M. ZERR, A. M., Ph. D.**, Professor of Mathematics and Science, Chester High School, Chester, Pa.; **GEORGE LILLEY, Ph. D., L. L. D.**, Professor of Mathematics. State University, Eugene, Ore.; **WALTER H. DRANE, A. M.**, Graduate Student, Harvard University, Cambridge, Mass.; and **ELMER SCHUYLER**, Reading, Pa.

The intrinsic equation to the logarithmic spiral $s=k(c^t-1)$.

$ds/dt=kc^t\log c$, for the evolute $\sigma=\pm(ds/dt)]_o^t$.

$\therefore\ \sigma=kc^t\log c-k\log c=k\log c(c^t-t)$.

$\therefore\ \sigma=k'(c^t-1)$, an equal spiral.

II. Solution by **J. SCHEFFER. A. M.**, Hagerstown, Md.; and **COOPER D. SCHMITT, A. M.**, Professor of Mathematics, University of Tennessee, Knoxville, Tenn.

Let P be a point of the given curve $r=a^\theta$, O the center of curvature, PQ a tangent at P, $PO=\rho$, $SQ=p=$the perpendicular from S upon PQ, $SP=r$, $SO=r'$, SM perpendicular to OP and$=p'$.

The pedal equation of the given curve $r=a^\theta$ is $r=p\sqrt{[1+(\log a)^2]}$; we also have $r'^2=\rho^2+r^2-2\rho p$, but $\rho=r\sqrt{[1+(\log a)^2]}$.

$\therefore\ r'=r\log a$, and since $p'^2=r^2-p^2$, we have

$$p'^2=\frac{r^2(\log a)^2}{1+(\log a)^2}.\quad \therefore\ p'=\frac{r\log a}{\sqrt{[1+(\log a)^2]}}.\quad \therefore\ p'=\frac{r'}{\sqrt{[1+(\log a)^2]}},$$

or, $r'=p'\sqrt{[1+(\log a)^2]}$, which is the pedal equation of the evolute and exactly like the pedal equation of the logarithmic spiril.

III. Solution by **CHAS. E. MYERS**, Canton, Ohio; and **P. H. PHILBRICK, M. S., C. E.**, Chief Engineer for Kansas City, Watkins & Gulf Railway Co., Lake Charles, La.

Let $r=$the radius vector of the given curve, $p=$the perpendicular on the tangent, $r_1=$the radius vector of the evolute, $p_1=$the perpendicular on its tangent, and $R=$the radius of curvature.

We have for the curve, $r=cp$.......(1).

$\therefore\ dr/dp=c$, and $R=cr$......(2).

$$r^2=p^2+p_1^2 \ .....(3). \quad r_1^2=R^2+r^2-2Rp......(4).$$

From (1), (2), and (4), we have,

$$r_1^2=c^2p^2(1+c^2)-2c^2p^2=c^2p^2(c^2-1)......(5).$$

From (3), and (1),

$$c^2p^2=p^2+p_1^2, \text{ or, } p^2(c^2-1)=p_1^2......(6).$$

From (5), and (6),

$$r_1^2=c^2p_1^2, \text{ or, } r_1=cp_1,$$

the equation of a similar and equal logarithmic spiral.

IV. Solution by J. W. YOUNG, Graduate Student, Ohio State University, Columbus, Ohio.

The equation to the logarithmic spiral is

$$r=ae^{m\theta}.$$

The equation to the normal at the point (r_1, θ_1) on the spiral is

$$r\cos[\theta-(\theta_1+\phi)]=r_1\cos\phi......(1),$$

where ϕ is the angle between r_1 and the tangent, and ϕ is constant, a known property of the spiral.

Expanding the left-hand member of (1), we have

$$r[\cos\theta(\cos\theta_1\cos\phi-\sin\theta_1\sin\phi)+\sin\theta(\sin\theta_1\cos\phi+\cos\theta_1\sin\phi)=r_1\cos\phi.$$

Dividing through by $\cos\phi$, and remembering that $\tan\phi$=constant, which can easily be shown to be $1/m$, we have

$$r\{\cos\theta[\cos\theta_1-(1/m)\sin\theta_1]+\sin\theta[\sin\theta_1+(1/m)\cos\theta_1]=r_1=ae^{m\theta_1}.$$

The required evolute is the envelope of this line where θ_1 is the parameter. Hence, arranging in terms of functions of θ_1, we have

$$r\{\cos\theta_1[\cos\theta+(1/m)\sin\theta]+\sin\theta_1[\sin\theta-(1/m)\cos\theta]\}=ae^{m\theta}......(2).$$

Differentiating with respect to θ_1,

$$r\{-\sin\theta_1[\cos\theta+(1/m)\sin\theta]+\cos\theta_1[\sin\theta-(1/m)\cos\theta]\}=ame^{m\theta_1}......(3).$$

We must eliminate θ_1 between (2) and (3).

Dividing (2) by (3), we have

$$\frac{\cos\theta_1(m\cos\theta+\sin\theta)+\sin\theta_1(m\sin\theta-\cos\theta)}{-\sin\theta_1(m\cos\theta+\sin\theta)+\cos\theta_1(m\sin\theta-\cos\theta)}=\frac{1}{m}.$$

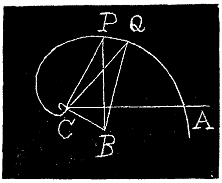
Reducing, $\cos\theta_1.\cos\theta=-\sin\theta_1\sin\theta$, or

$$\frac{\cos\theta_1}{\sin\theta}=-\frac{\sin\theta_1}{\cos\theta}=\frac{\sqrt{(\cos^2\theta_1+\sin^2\theta_1)}}{\sqrt{(\sin^2\theta+\cos^2\theta)}}=1.$$

Hence, $\cos\theta_1=\sin\theta$, $\sin\theta_1=-\cos\theta$.
$\therefore\ \theta_1=\theta-\frac{1}{2}\pi$.
$\therefore$ The required equation is $r=ae^{m(\theta-\frac{1}{2}\pi)}=a'e^{m\theta}$.

V. **Solution by P. S. BERG, B. Sc., Principal of Schools, Larimore, N. D.**

Let P and Q be two points on the curve, C its pole, PB, QB, the normals at P and Q; join CB. Then the angles CPB and CQB are equal, and consequently the four points C, P, Q, B, lie on a circle. Hence $\angle QCB=\angle QPB$; but in the limit when P and Q are coincident, the angle QPB becomes a right angle and B becomes the center of curvature belonging to point P; hence PCB also becomes a right angle, and the point B is determined.



Again, $\angle CBP=\angle CQP$; but, in the limit, the angle CQP is constant; therefore angle CBP is also constant, and since the line BP is a tangent to the evolute at B, it follows that the tangent makes a constant angle with the radius vector CB. From this property it follows that the evolute in question is another logarithmic spiral.

Again, as the constant angle is the same for the curve and for its evolute, it follows that the latter curve is the same spiral.

Professors M. C. Stevens and Elmer Schuyler refer to the solution in *Williamson's Differential Calculus*, page 300.

MECHANICS.

NOTE ON PROBLEM 82.

Professor Zerr, in his solution of this problem, assumed limiting friction at all points. Of course there is limiting friction between box and floor. But limiting friction does not in general exist at both the other two points; for suppose it did, then the sphere in its descent will not revolve about its center.

Hence, taking moments about the center (using his notation) we have

$$\tfrac{1}{3}W'=\frac{2W'}{3\cos\theta+9\sin\theta}.$$

Whence $\theta=60°$, no matter what the relation between W and W'. Of